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aggregation (production)

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Abstract

Aggregation concerns the conditions under which several variables can be treated as one, or macro-relationships derived from micro-relationships. This problem is especially important in production, where, without proper aggregation, one cannot interpret the properties of the aggregate production function. The conditions under which aggregate production functions exist are so stringent that real economies surely do not satisfy them. The aggregation results pose insurmountable problems for theoretical and applied work in fields such as growth, labour or trade. They imply that intuitions based on micro variables and micro production functions will often be false when applied to aggregates.

Keywords

aggregation (production); Cambridge capital theory debates; capital aggregation; Cobb–Douglas functions; endogenous growth; growth accounting; Hicks, J.; Hicks–Leontief aggregation; labour aggregation; Leontief, W.; National Income and Products Account (NIPA); neoclassical growth theory; output aggregation; production functions; productivity (measurement problems); total factor productivity

Article

Aggregation in production concerns the conditions under which macro production functions can be derived from micro production functions. Microeconomic theory elegantly treats the behaviour of optimizing individual agents in a world with an arbitrarily long list of individual commodities and prices. However, the desire to analyse the great aggregates of macroeconomics – gross national product, inflation, unemployment, and so forth – leads to theories that treat such aggregates directly. The aggregation 'problem' matters because without proper aggregation one cannot interpret the properties of such macroeconomic models. This is particularly true as regards the production sector.

Leontief's theorem

Underlying many results on aggregation is a theorem of Leontief (1947a; 1947b). Let x and y be vectors of variables and $F(x, y)$ a twice-differentiable function. It is desired to aggregate over x, that is, to replace x with a scalar aggregator function, $g(x)$, such that $F(x) = H[g(x), y]$. This can be done if and only if, along any surface on which $F(x, y)$ is constant, the marginal rate of substitution between each pair of elements of x is independent of y. (For a proof, see Fisher, 1993, pp. xiv–xvi.)

Hicks–Leontief aggregation

Since optimizing, price-taking agents equate marginal rates of substitution to price ratios, one restriction permitting aggregation over commodities is the assumption that the prices of all goods to be included in an aggregate always vary proportionally. This is called 'Hicks–Leontief aggregation' (Leontief, 1936; Hicks, 1939) and is a powerful expository tool. It requires no special assumptions as to the form of utility or production functions, but is applicable only in relatively artificial situations. Under more general circumstances, restrictions on utility or production functions become essential.

Aggregation in consumption

Consider a single household. Suppose that we wish to describe behaviour in terms of aggregate commodities such as 'food' or 'clothing'. By Leontief's Theorem, a food aggregate exists if and only if the marginal rate of substitution between any two kinds of food is independent of consumption of any non-food commodity. If a similar restrictive condition is satisfied for all the aggregates to be constructed, then the household's utility function can be written in aggregate terms.

Even such restrictive conditions will not always suffice. If we wish to represent the household as maximizing the aggregate utility function subject to an aggregate budget constraint, we must have aggregate prices as well as aggregate consumption goods. This requires that aggregates such as 'food' be homothetic in their component variables, again considerably restricting the household's utility function (Gorman, 1959; Blackorby et al., 1970).

Aggregation over agents presents a different set of questions. Suppose that we wish to treat the aggregate demands of a collection of households as the demands of a single, aggregate household. Then, only aggregate income and not its distribution can influence demand. At given prices, this makes the income derivative of every household's demand for a given commodity the same constant. Engel curves must be parallel straight lines. If zero income implies zero consumption, then all households must have the same homothetic utility function (Gorman, 1953).

In general, the only consumer-theoretic restrictions obeyed by aggregate demand functions are those of continuity, homogeneity of degree zero, and the various restrictions implied by the budget constraint (cf. Sonnenschein, 1972; 1973).

Aggregation in production

A more detailed survey of much of what follows in this section is given in Felipe and Fisher (2003). The analysis of aggregation conditions for production functions is far richer and the conditions even more demanding than in the case of demand functions. Moreover, the subject has a complicated history and bears on the very foundations of neoclassical *macro*economics, negatively implicating the use of such important concepts as 'total factor productivity', 'natural rate of growth', 'capital–labour ratio', and even such terms as 'investment', 'capital', 'labour', and 'output'.

To take a simple example, suppose we have two production functions $Q^A = f^A(K_1^A, K_2^A, L^A)$ and $Q^B = f^B(K_1^B, K_2^B, L^B)$ for firms *A* and *B*, where $K_1 = K_1^A + K_1^B$, $K_2 = K_2^A + K_2^B$ and $L = L^A + L^B$(*K* refers to capital – two types – and *L* to labour – assumed homogeneous). The problem is to determine whether and in what circumstances there exists a function $K = h(K_1, K_2)$ where the aggregator function $h(\cdot)$ has the property that $G(K, L) = G[h(K_1, K_2), L] = \Psi(Q^A, Q^B)$, and the function Ψ is the production possibility curve for the economy. Note that we have implicitly assumed that a production function exists for the firm. Further, even within the firm there is a problem of aggregation over factors. Here, we concentrate on aggregation over firms.

Klein (1946a; 1946b) initiated the first debate on aggregation in production functions. He argued that the aggregate production function should be strictly a technical relationship, akin to the micro production function, and objected to utilizing the entire micro model with the assumption of profit-maximizing behaviour by producers in deriving the production functions of the macro model. However, Kenneth May (1947) pointed out that this program is not generally achievable and, indeed, rests on a misunderstanding of what production functions actually are – even at the micro level. A production function does not tell us what outputs are or can be produced from a given set of inputs. It tells us what the *maximum* output is of a particular commodity, given a vector of inputs and the other outputs that are also to be produced from them.

That Klein's aggregation program is generally unachievable was specifically proved by André Nataf (1948). He showed that such aggregation is possible if and only if all micro production functions are additively separable in capital and labour.

The problem here is as follows. Suppose there are n firms indexed by $\nu = 1, \ldots, n$. Each produces the same output $Y(\nu)$ using the same type of labour $L(\nu)$, and a single type of capital $K(\nu)$. The νth firm has a two-factor production function $Y(\nu) = f^{\nu}\{K(\nu), L(\nu)\}$. The total output of the economy is $Y = \sum_{\nu} Y(\nu)$, total labour is $L = \sum_{\nu} L(\nu)$. Capital, on the other hand, may differ from firm to firm. Under what conditions can total output Y be written as $Y = \sum_{\nu} Y(\nu) = F(K, L)$ where $K = K\{K(1), \ldots, K(n)\}$ and $L = L\{L(1), \ldots, L(n)\}$ are indices of aggregate capital and labour, respectively? Nataf showed that, where the variables $K(\nu)$ and $L(\nu)$ are free to take on all values, the aggregate production function $Y = F(K, L)$ exists, if *and only if* every firm's production function is additively separable in labour and capital, that is, if every f^{ν} can be written in the form $f^{\nu}\{K(\nu), L(\nu)\} = \varphi^{\nu}\{K(\nu)\} + \psi^{\nu}\{L(\nu)\}$. Moreover, if one insists that labour aggregation be 'natural', with the L appearing in the aggregate production function, then all the $\psi^{\nu}\{L(\nu)\} = c\{L(\nu)\}$, where c is the same for all firms.

Nataf's theorem provides an extremely restrictive condition for inter-sectoral or even inter-firm aggregation. Evidently, aggregate production functions will not exist unless there are some further restrictions on the problem.

In fact, such restrictions are available; they stem from the requirement that a production function describe *efficient* production possibilities.

Capital aggregation

Consider the simplest case of two factors, with physically homogeneous capital (K) and homogeneous labour (L), where total capital can be written as $K = \sum_{\nu} K(\nu)$, efficient production requires that aggregate output Y be maximized given aggregate labour (L) and aggregate capital (K). Under these simplified circumstances, it follows that $Y^{M} = F(K, L)$ where Y^{M} is maximized output, since, as was pointed out by May (1946; 1947), individual allocations of labour and capital to firms would be determined in the course of the maximization problem. This holds even if all firms have different production functions and whether or not there are constant returns.

In the (somewhat) more realistic case where only labour is homogeneous and technology is embodied in capital, Fisher (1965) proposed to treat the problem as one of labour being allocated to firms so as to maximize output, with capital being firm-specific. Here, no 'natural' aggregate of capital exists.

Given that output is maximized with respect to the allocation of labour to firms, with such maximized output denoted by Y^{*}, the question becomes: under what circumstances is it possible to write total output as $Y^{*} \ F(J, L)$ where $J = J\{K(1), \ldots, K(n)\}$, where $K(\nu)$, $\nu = 1, \ldots, n$, represents the stock of capital of each firm (that is, one kind of capital per firm)? Since the values of $L(\nu)$ are determined in the optimization process there is no labour aggregation problem. The entire problem in this case lies in the existence of a capital aggregate. Since Leontief's condition is both necessary and sufficient for the existence of a group capital index, the previous expression for Y^{*} is equivalent to $Y^{*} = G\left\{K(1), \ldots, K(n), L\right\}$ if and only if the marginal rate of substitution between any pair of the $K(\nu)$ is independent of L.

Fisher drew the implications of this condition. He showed that, under strictly diminishing returns to labour ($f^{\nu}_{LL} < 0$), if any one firm has an additively separable production function (that is, $f^{\nu}_{KL} \equiv 0$), then a necessary and sufficient condition for capital aggregation is that *every* firm have such a production function. (Throughout, such subscripts denote partial differentiation in the obvious manner.) This means that capital aggregation is impossible if there is both a firm which uses labour and capital in the same production process, and another one which has a fully automated plant. Fisher found that a necessary and sufficient condition for capital aggregation is that every firm's production function satisfy a partial differential equation in the form $f^{v}_{KL} / f^{v}_{K} f^{v}_{LL} = g(f^{v}_{L})$, where g is the same function for all firms. More important, on the assumption of constant returns to scale, the case of capital-augmenting technical differences (that is, embodiment of new technology can be written as the product of the amount of capital times a coefficient) turns out to be the *only case* in which a capital aggregate exists. This means that each firm's production function must be writeable as $F(b_v K_v, L_v)$, where the function $F(\cdot, \cdot)$ is common to all firms, but the parameter b_v can differ. Under these circumstances, a unit of one type of new capital equipment is the exact duplicate of a fixed number of units of old capital equipment ('better' is equivalent to 'more'). As we would expect, given constant returns to scale, the aggregate stock of capital can be constructed with capital measured in efficiency units. Fisher (1965) could not come up with a closed-form characterization of the class of cases in which an aggregate stock of capital exists when the assumption of constant returns is dropped. Nevertheless, as he showed, there do exist classes of non-constant returns production functions which do allow construction of an aggregate capital stock. On the other hand, if constant returns are not assumed there is no reason why perfectly well-behaved production functions cannot fail to satisfy Fisher's partial differential equation given above. Capital aggregation is then impossible if any firm has one of these 'bad apple' production functions. To sum up: aggregate production functions exist if and only if all micro production functions are identical except for the capital efficiency coefficient – an extremely restrictive condition.

Working with the profits function rather than with the production function, Gorman (1968) reached similar conclusions to those of Fisher.

Fisher extended his original work. First of all, he analysed (1965) the case where each firm produces a single output with a single type of labour, but two capital goods, that is, $Y(v) = f^{v}(K_1, K_2, L)$. Here Fisher distinguished between two different cases. The first is that of aggregation across firms over one type of capital (for example, plant or equipment). Fisher concluded that the construction of a sub-aggregate of capital goods requires even more stringent conditions than for the construction of a single aggregate. For example, if there are constant returns in K_1, $K_2$and L, there will not be constant returns in $K_1$and L, so that the difficulties of the two-factor non-constant returns case appear. Further, if the vth firm has a production function with all three factors as complements, then no K_1 aggregate can exist. Thus, for example, if any firm has a generalized Cobb–Douglas production function (with the v argument omitted) in plant, equipment, and labour $Y = AK_1^{\alpha} K_2^{\beta} L^{1-\alpha-\beta}$, one cannot construct a separate plant or separate equipment aggregate for the economy as a whole (although this does not prevent the construction of a full capital aggregate).

The other case Fisher (1965) considered was that of the construction of a complete capital aggregate. In this case, a necessary condition is that it be possible to construct such a capital aggregate for each firm taken separately; and a necessary and sufficient condition (with constant returns), given the existence of individual firm aggregates, is that all firms differ by at most a capital augmenting technical difference. They can differ *only* in the way in which their individual capital aggregate is constructed.

Second, Fisher (1982) asked whether the crux of the aggregation problem derives from the fact that capital is considered to be an immobile factor. He showed that the aggregation problem seems to be due only to the fact that capital is fixed and is not allocated efficiently. That is true in the context of a two-factor production function. However, if one works in terms of many factors, all mobile over firms, and asks when it is possible to aggregate them into macro groups, the mobility of capital has little bearing on the issue. In fact, where there are several factors, each of which is homogeneous, optimal allocation across *firms* does not guarantee aggregation across *factors*. The conditions for the existence of such aggregates are still very stringent, but this has to do with the necessity of aggregating over firms rather than with the immobility of capital. A possible way of interpreting the existence of aggregates at the firm level is that each firm could be regarded as having a two-stage production process. In the first one, the factors to be aggregated, $X_i( )$, are combined to produce an intermediate output, $\varphi^{v}(X(v))$. This intermediate output is then combined with the other factor, $L(v)$, to produce the final output. Aggregation of X can be done if and only if firms are either all alike as regards the first stage of production, or all alike as regards the second stage. If they are all alike as regards the first stage, then the fact that L is mobile plays no role. If they are all alike as regards the second stage, then the fact that the x_i are mobile plays no role.

Finally, Fisher (1983) is another extension of the original problem to study the conditions under which full and partial capital aggregates, such as 'plant' or 'equipment', would exist simultaneously. Not surprisingly, the results are as restrictive as those above. See also Blackorby and Schworm (1984).

Labour and output aggregation

Fisher (1968) went on to study the problems involved in labour and output aggregation, pointing out that the aggregation problem is not restricted to capital. Output aggregation and labour aggregation are also necessary if one wants to use a sector-wide or economy-wide aggregate production function.

Fisher again studied aggregation over firms, with labours and outputs shifted over firms to achieve efficient production, given the capital stocks. In the simplest case of constant returns, a labour aggregate will exist if and only if a given set of relative wages induces all firms to employ different labours in the same proportions. Similarly, where there are many outputs, an output aggregate will exist if and only if a given set of relative output prices induces all firms to produce all outputs in the same proportion. Thus, the existence of a labour aggregate requires the absence of specialization in employment; and the existence of an output aggregate requires the absence of specialization in production – indeed, all firms must produce the same market basket of outputs differing only in their scale. (Blackorby and Schworm, 1988, is an extension of Fisher, 1968.)

Houthakker–Sato aggregation conditions

Whereas Fisher sought to develop conditions where aggregate production functions would always work, Houthakker (1955–56) and Sato (1975) considered two-factor cases in which the problem was restricted by assuming that the distribution of capital over firms remains constant. In such cases it is obvious that one can aggregate over capital. Houthakker and Sato's contributions (see also Levhari, 1968) were to show the relationships between the fixed distribution of capital and the form of the aggregate production function.

Fisher's simulations

But, if aggregate production functions do not exist, how is it that they appear to 'work' in the sense that they fit the data well, that the estimated elasticities are close to the factor shares, and that wage rates are approximate the calculated marginal product of labour? We shall have more to say on this below, but here consider another result of Fisher (1971). This paper reports the results of simulations in a simple (heterogeneous capital, homogeneous labour and output) economy in which the aggregation conditions are known not to be satisfied. The principal result is that when, despite this, calculated factor shares just happen to be roughly constant, then the Cobb–Douglas aggregate production function 'works' in the above sense, even though the approximate constancy of factor shares *cannot* be caused by the non-existent aggregate production function. (See Fisher, Solow and Kearl, 1977 for the case of the CES production function.)

Implications for empirical work

Empirically, the non-existence of the aggregate production function poses a conundrum. If aggregate production functions do not exist, there must be some other reason why they seem to work empirically. The answer has been in the literature for a long time (Simon and Levy, 1963; Simon, 1979; Shaikh, 1980), and more recently Felipe (2001) and Felipe and McCombie (2001; 2002; 2003; 2005; 2006a;

2006b) have elaborated upon it. (For an in-depth discussion of these issues see the papers in the *Eastern Economic Journal*, 2005.) However, like the theoretical arguments underlying the non-existence of the aggregate production function, these arguments have largely been ignored.

The argument is that, because the data used in aggregate empirical applications are not physical quantities but values, the accounting identity that relates definitionally the value of total output to the sum of the value of total inputs can be rewritten as a form that resembles a production function.

More specifically, the National Income and Products Account (NIPA) identity states that value added equals the wage bill plus total profits, that is,

$$V_t \equiv W_t + \Pi_t \equiv w_t L_t + r_t J_t$$
(1)

where V is real value added, W is the total wage bill in real terms, Π denotes total profits ('operating surplus', in the NIPA terminology), also in real terms, w is the average real wage rate, L is employment, r is the average *ex post* real profit rate, and J is the deflated or constant-price value of the stock of capital. (Expression (1) is an accounting identity, not the result of Euler's Theorem.) In applied aggregate work, the measures of output and capital used are the constant-price values, not physical quantities. We denote them by V and J, respectively. These are different from Y and K used above, which denoted physical quantities. The symbol ≡ indicates that expression (1) is an accounting identity.

Expressing the identity (1) in growth rates yields:

$$\hat{V}_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t + a_t \hat{L}_t + (1-a_t)\hat{J}_t$$
(2)

where ^ denotes a proportional growth rate, $a_t \equiv w_t L_t / V_t$ is the share of labour in output, and $1-a_t \equiv r_t J_t / V_t$ is the share of capital. So far no assumption of any kind has been made.

Suppose now that factor shares in the economy are relatively stable. This could be due, for example, to the fact that firms set prices according to a mark-up on unit labour costs. Assume also that w_t and r_t grow at constant rates. Then

$$\hat{V}_t \equiv \lambda + a\hat{L}_t + (1-a)\hat{J}_t$$
(3)

where $\lambda \equiv a\hat{w} + (1-a)\hat{r}$. Integrating (3) and taking antilogarithms,

$$V_t \equiv A_0 \exp(\lambda t) L_t^{a} J_t^{1-a}$$
(4)

Expression (4) is simply the NIPA accounting identity, expression (1), rewritten under the two assumptions mentioned above. It is certainly not a Cobb–Douglas production function, as such does not exist.

What are the implications of this argument? Suppose one estimates the standard Cobb–Douglas regression $V_t = C_0 \exp(\gamma t) L_t^{\alpha_1} J_t^{\alpha_2}$ and in this economy factor shares are approximately constant and wage and profit rate growth is approximately constant. Then, this regression will yield very good results, since it approximates the identity (4). The statistical fit will be close to unity, $\alpha_1 \cong a$, $\alpha_2 \cong 1-a$, and $\gamma \cong \lambda$. However, the aggregate production function may not exist, or firms in this economy may be subject to increasing returns to scale, although the regression results might lead us to believe otherwise.

On the other hand, if the assumptions about the path of the factor shares and the growth rates of w and r are incorrect, the regression $V_t = C_0 \exp(\gamma t) L_t^{\alpha_1} J_t^{\alpha_2}$ will not yield good results. Felipe and Holz, 2001, showed using Monte Carlos simulations that the main reason why the Cobb–Douglas regression $V_t = C_0 \exp(\gamma t) L_t^{\alpha_1} J_t^{\alpha_2}$ often fails is that the approximation of $[a_t \hat{w}_t + (1-a_t)\hat{r}_t]$ through the constant term λ is incorrect. Such widely discussed problems as unit roots or endogeneity of the regressors are not the key issues. This simply means that we have to search for better approximations to the identity. (See Felipe and McCombie, 2001; 2003, for the derivations of the CES and translog approximations to the accounting identity.)

These results have devastating implications for empirical neoclassical macro growth theory, including endogenous growth, and total factor productivity measurement and growth accounting exercises. Indeed, Felipe and McCombie (2006b) have shown using simulations that the true rate of technical progress, computed with the use of firm-level data, is very different from that obtained with the use of aggregate data. Indeed, the two measures of productivity are so far apart that it is concluded that total factor productivity growth calculated with aggregate data is in no way a proxy for the true rate of technological progress.

Why do economists continue using aggregate production functions?

Most economists are not aware of these results, but simply think of the aggregate production function as part of their basic toolkit. Others use such concepts as total productivity growth without realizing that they are assuming the existence of a non-existent construct.

Some economists, on the other hand, are aware of the aggregation results and yet continue using aggregate production functions. The reasons for doing so fall under three broad categories:

1. 1. Aggregate production functions are seen as useful parables (Samuelson, 1961–62).
2. 2. So long as aggregate production functions appear to give empirically reasonable results, why shouldn't they be used?
3. 3. For the applications where aggregate production functions are used, there is no other choice.

However, in the light of the aggregation results, none of these reasons seems valid.

Samuelson's parable argument was stated in the context of the so-called Cambridge capital theory debates. (It should not be thought that the aggregation problems have no bearing on the Cambridge–Cambridge debates. The discovery that aggregate production functions can violate properties that one expects of production functions, so-called reswitching and reverse capital-deepening, was at bottom a discovery that the aggregate concept used is not a production function at all. The aggregation problem literature shows that this was to be expected.) Samuelson showed that even in cases with heterogeneous capital goods some rationalization could be provided for the validity of the neoclassical parable, which assumes that there is a single homogenous factor referred to as capital, whose marginal product equals the interest rate. But Samuelson's results hold only in very restrictive cases, as we should expect from the aggregation literature. (See also Garegnani, 1970.)

A variation of the parable argument is that the aggregate production function should be understood as an *approximation*. It is evident that Fisher's (exact) aggregation conditions are so stringent that one can hardly believe that actual economies will satisfy them even approximately. Fisher (1969), therefore, asked: What about the possibility of a *satisfactory approximation*? Thus, suppose the values of capitals and labours in the economy lie in a bounded set and the requirement is that an aggregate production function lie within some specified distance of the true production surface for all points in the bounded set. Can this happen without the approximate satisfaction of the aggregation conditions? Fisher showed that this cannot reasonably happen by proving that the *only* way for approximate aggregation to hold without approximate satisfaction of the Leontief conditions is for the derivatives of the functions involved to wiggle violently up and down, an unnatural property not exhibited by the aggregate production functions used in practice.

The second argument is that, despite the aggregation results, neoclassical macroeconomic theory generally deals with macroeconomic aggregates derived by analogy with the micro concepts. Then, the argument goes, why not continue using them? Naturally, the aggregation problem appears in all areas of economics, including consumption theory, where a well-defined micro consumption theory exists. The neoclassical aggregate production function is also built by analogy (Ferguson, 1971).

This argument is untenable. Employing macroeconomic production functions on the unverified premise that inference by analogy is correct is inadmissible. Further, as opposed to the (already suspect) case of

the consumption function, the conditions for successful aggregation of production functions seem far more outlandish.
The third and final argument given for the use of aggregate production functions is that there is no other option if one is to answer the questions for which the aggregate production function is used, for example to discuss productivity differences across nations. But, 'It's crooked, but it's the only wheel in town' is not a scientific argument. The profession needs to find a different 'wheel'.

See Also

- aggregation (theory)
- cost functions
- endogenous growth theory
- growth accounting
- neoclassical growth theory
- production functions
- total factor productivity

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